Int. J. Heat Mass Transfer. Vol. 12, pp. 439–443. Pergamon Press 1969. Printed in Great Britain

HEAT TRANSFER COEFFICIENTS AND THERMAL CONDUCTIVITY OF COLD LIQUIDS†

DAVID N. FRENCH

Abex Corporation, Research Center, Mahwah, New Jersey

CLYDE M. ADAMS, Jr. ‡

Professor of Metallurgy, Massachusetts Institute of Technology, Cambridge, Massachusetts

Abstract—A method for measuring the thermal conductivity of a low temperature liquid and the heat-transfer coefficient between a freezing ice sphere and the low temperature liquid is presented. The calculations are based on equating the quantity of heat liberated by a freezing ice sphere to the amount of heat that crosses the ice-"mold" interface. By suitably handling the directly measurable variables of sphere diameter and freezing times, the values of thermal conductivity and heat transfer coefficient can be obtained from slopes and intercepts of straight lines.

NOMENCLATURE

- A, area of spherical cavity [in.^{2}];
- C_p, heat capacity of the "mold" [Btu/lb°F];
- D, spherical cavity diameter [in];
- h, coefficient of heat transfer [Btu/in.^{2}min°F];
- H, heat of fusion of water [142 Btu/lb];
- K, thermal conductivity of "mold" [Btu/in. min°F];
- Q, quantity of heat [Btu];
- T_m, freezing point of ice [32°F];
- T_0, initial temperature of the "mold", −4°F;
- T_s, temperature of the ice-"mold" interface [°F];
- V, spherical cavity volume [in.^{3}];
- α, thermal diffusivity [in.^{2}/min], $(K/\rho C_p)$;
- ρ, density of the water, 0·036 lb/in.^{3};
- ρ', density of the "mold", 0·033 lb/in.^{3};
- θ_f, freezing time, min.

INTRODUCTION

THE FOLLOWING paper presents a technique for measuring the thermal conductivity of a liquid in the temperature range 0 to −40°F. The calculations are based on equating the quantity of heat liberated by a freezing ice sphere to the amount of heat that crosses the ice-"mold" interface. By suitably handling the directly measurable variables of sphere diameter and freezing time, the values of the thermal conductivity and heat-transfer coefficient can be obtained from slopes and intercepts of straight lines.

THEORY

Consider an infinite medium of initial temperature T_0 surrounding an internal spherical cavity of diameter D. At zero time, the temperature of the cavity is raised to a constant temperature T_m. Assume the interface has a constant temperature T_s. T_s may be thought of as a time averaged value. The amount of heat Q that enters the "mold" is [1]:

$$Q = AK(T_s - T_0)\left[\frac{2\theta_f}{D} + \frac{2\sqrt{\theta_f}}{\sqrt{\pi\alpha}}\right]. \qquad (1)$$

(See nomenclature list for definition of symbols used.) The same quantity of heat must have crossed the boundary surface of the cavity and is given by [1]:

$$Q = Ah(T_m - T_s)\,\theta_f\,. \qquad (2)$$

† Work reported here performed while the authors were associated with the Massachusetts Institute of Technology.

‡ Present Address, Professor of Metallurgical Engineering, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin.

Equations (1) and (2) may be combined to eliminate the unknown interface temperature T_s:

$$\frac{A}{Q}(T_m - T_0) = \frac{1}{h\theta_f} + \frac{1}{\dfrac{2K\theta_f}{D} + \dfrac{2K\sqrt{\pi\theta_f}}{\sqrt{\pi\alpha}}}. \quad (3)$$

If the cavity is pure water at its freezing point and only the heat of fusion is considered, Q is then:

$$Q = V\rho H. \quad (4)$$

For a sphere:

$$\frac{V}{A} = \frac{D}{6}. \quad (5)$$

Combining equations (3), (4) and (5):

$$\frac{\theta_f}{D} = \frac{\rho H}{6(T_m - T_0)\,h} + \frac{\rho H}{12(T_m - T_0)\,K} \times \left[\frac{1}{\dfrac{1}{D} + \dfrac{1}{\sqrt{(\pi\alpha\theta_f)}}}\right] \quad (6)$$

Let

$$Z = \frac{\rho H}{6(T_m - T_0)\,h} \quad (7)$$

and

$$Y = \frac{\rho H}{12(T_m - T_0)\,K} \quad (8)$$

From equations (6), (7) and (8), a quadratic equation in D may be found:

$$D^2[Z + Y\sqrt{(\pi\alpha\theta_f)}] + D[Z\sqrt{(\pi\alpha\theta_f)} - \theta_f] - \theta_f\sqrt{(\pi\alpha\theta_f)} = 0 \quad (9)$$

solving for D:

$$D = \frac{[\theta_f - Z\sqrt{(\pi\alpha\theta_f)}] + \{[Z\sqrt{(\pi\alpha\theta_f)} - \theta_f]^2 + 4Z\theta_f\sqrt{(\pi\alpha\theta_f)} + 4Y\pi\alpha\theta_f^2\}^{\frac{1}{2}}}{2Z + 2Y\sqrt{(\pi\alpha\theta_f)}}. \quad (10)$$

Equation (10) may be simplified for values of θ_f greater than about four minutes. Under these conditions, Z may be neglected compared with $Y\sqrt{(\pi\alpha\theta_f)}$, $Z\sqrt{(\pi\alpha\theta_f)}$ may be neglected compared with θ_f, and only the θ_f^2 terms inside the brackets are of significance. Thus:

$$D \approx \frac{\theta_f^{\frac{1}{2}}[1 + (1 + 4Y\pi\alpha)^{\frac{1}{2}}]}{2Y\sqrt{\pi\alpha}}. \quad (11)$$

A plot of D vs. $\theta_f^{\frac{1}{2}}$ will be linear and the slope will be:

$$\text{slope} = \frac{1 + (1 + 4\pi Y\alpha)^{\frac{1}{2}}}{2Y\sqrt{\pi\alpha}} \quad (12)$$

From equation (12), the value of K is determined.

A plot of equation (6) in the form

$$\frac{\theta_f}{D} \text{ vs. } \frac{1}{\dfrac{1}{D} + \dfrac{1}{\sqrt{(\pi\alpha\theta_f)}}} \quad (6a)$$

should be a linear function with the slope proportioned to 1/K and intercept proportional to 1/h.

EXPERIMENTAL PROCEDURES

The "mold" into which the ice spheres were "cast" was a large beaker of organic liquid at −4°F. It was necessary to use a clear fluid so that the thermocouple junction could be seen easily to facilitate centering the sphere on the junction. A liquid that is neither dissolved by nor dissolves in water was required; otherwise, the sphere would disappear or change composition and thus its freezing point. The liquid chosen for the "mold" was a mixture of 20 volume per cent kerosene and 80 volume per cent mineral oil, which has a high enough viscosity to prevent the sphere from sinking too rapidly during freezing. The surface tension of water tended to make the drops spherical but the density difference between "mold" and water caused the drops to be slightly elliptical. In all cases the thermocouple held the drop stationary so that freezing was accomplished in a spherically symmetrical environment. Water at its freezing point was injected with an eye dropper onto the thermocouple junction. Drop sizes of about $\frac{1}{4}$–$\frac{1}{2}$ in dia. could conveniently be made with this simple procedure. Smaller sizes than $\frac{1}{4}$ in could not be centered on the thermocouple junction and sizes greater than $\frac{1}{2}$ in became quite elliptical and destroyed spherical symmetry.

For all temperature measurements, copper-constantan thermocouples were used and temperature continuously recorded on a single channel Brown instrument. Two thermocouple wire sizes were used, 4 and 20 mil, to study the influence of the wire size on the freezing time. In both cases, the thermocouple was used in the straight position with the junction in the center of the beaker. To prevent as much as possible the formation of air bubbles during freezing, the water was boiled for 10 minutes to exclude any dissolvea gases and stored while cooling in a closed contaıner.

Ice sphere diameters were measured on several diameters with a micrometer and the smallest measurement used as the diameter. If the difference between the largest and smallest measured diameter was more than 5 per cent, that particular determination was not used.

RESULTS

The results of the freezing time measurements are presented in Fig. 1. No significant difference is observed in the freezing times as measured by the two thermocouple wire sizes used (4 and 20 mm). These data are treated together. A least squares analysis of Fig. 1 for the best straight line gives:

$$D = 0{\cdot}173\,\theta_f^{\frac{1}{2}} - 0{\cdot}079. \tag{13}$$

Using equation (12) with the slope of 0·173 from equation (13) gives:

$$0{\cdot}173 = \frac{1 + (1 + 4\pi Y\alpha)^{\frac{1}{2}}}{2Y(\sqrt{\pi\alpha})}. \tag{14}$$

Noting that:

$$\alpha = \frac{K}{\rho' C_p}$$

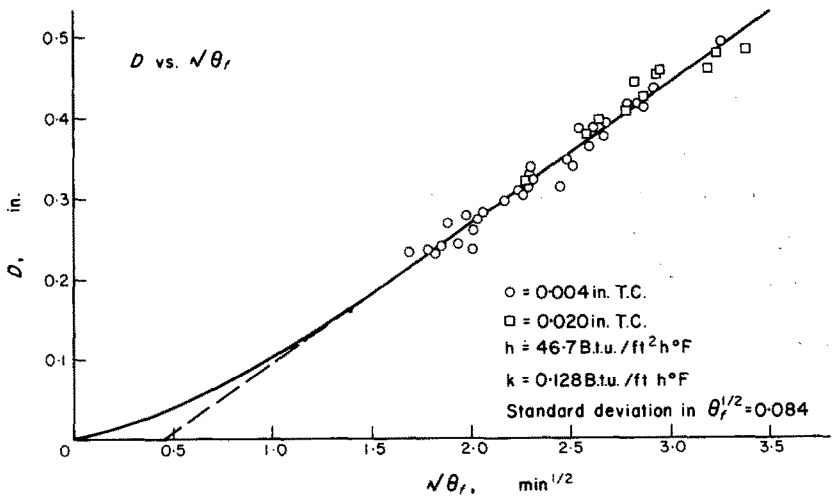

FIG. 1. A plot of D vs. $\sqrt{\theta_f}$ that shows the linear relationship observed, slope is proportional to K.

equation (14) may be solved for K, the thermal conductivity of the "mold". Using $\rho' = 0{\cdot}033$ lb/in^3, a measured quantity at -4°F, $C_p = 0{\cdot}5$ Btu/lb°F, a handbook value [2] for oils of this nature, $Y = 0{\cdot}0118/K$, from its definition and the constants given above with $T_m - T_0 = 36$°F, K is found to have a value of 0·13 Btu/ft h°F ($1{\cdot}8 \times 10^{-4}$ Btu/in min°F).

The magnitude of h is determined by using the intercept of equation (6) found in the following manner. The value of K just computed from equation (14) is used to compute the slope of equation (6). Next, the "center of mass" of the data in Fig. 2 is calculated, and this point plus the known slope of the line will give the intercept. The curve in Fig. 2 is given a slope corresponding to the K value of equation (14) and is forced to pass through the "center of mass" of the data. The limits of h are found by calculating the standard deviation of the data from the line used to find h and using the 95 per cent confidence limits, that is, two standard deviations on either side of the intercept.

The line through the "center of mass" of the data in Fig. 2 with the slope proportional to the reciprocal of the K value found from equation (14) is:

$$\frac{\theta_f}{D} = 4{\cdot}37 + \frac{66{\cdot}5}{\dfrac{1}{D} + \dfrac{1}{\sqrt{(\pi\alpha\theta_f)}}}. \tag{15}$$

Standard deviation in θ_f/D is 1·06. From the intercept value of 4·37, the value of h is 47 Btu/ft^2 hr°F. Taking the error in h to be twice the standard deviation on either side of 4·37, h varies from 32 to 92 Btu/ft^2 h°F.

The curved tail portion of Fig. 1 for θ_f less than about 4 min is computed from equation (10) using the experimental values of K and h. For θ_f greater than 4 min, the two curves are practically identical.

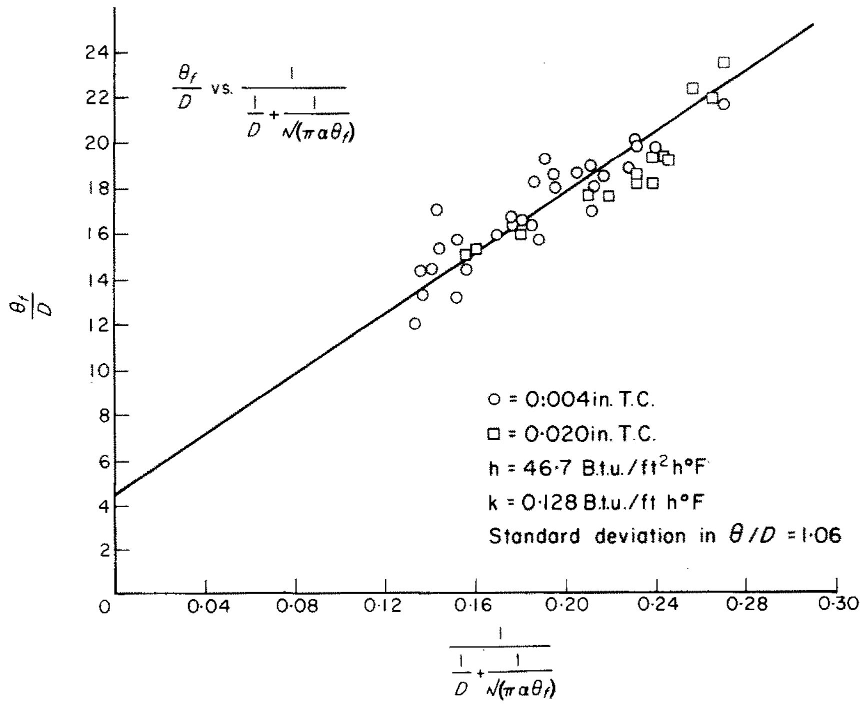

FIG. 2. A plot of θ_f/D vs. $[1/D + 1/\sqrt{(\pi\alpha\theta_f)}]^{-1}$ that shows the linear relationship observed, the slope is proportional to $1/K$ and the intercept is proportional to $1/h$.

DISCUSSION

In the measurement of the thermal conductivity of liquids, the major source of error lies in the contribution to the heat flux made by convection. For the most part, the current procedures [3] use either a downward flow of heat or thin films of the liquid with small temperature gradients, a few degrees at most. Both procedures reduce the convective heat transfer. The present method makes no provision for suppressing this effect and, therefore, is subject to error. The scatter in the data, both for the determination of k and h, reflect the variations that arise from turbulence in the liquid "mold".

Some idea of the accuracy of the measurement of k can be obtained by applying regression analysis to the data of Fig. 1. The 95 per cent confidence limits of the slope can be determined using statistical analysis methods [4]. From such an analysis, the slope is accurate within 5 per cent, and thus k is accurate only within 10 per cent. Published methods for determining k for liquids show accuracies within 2 or 3 per cent [3]. It seems that the present procedure gives, at best, an engineering approximation and may be more useful in determining heat transfer coefficients.

The measurements do not require elaborate equipment, only a temperature recorder, thermocouples, a refrigerator and a compatable freezing liquid and "mold". Values of k and h accurate within about 10 per cent can be obtained in a few hours time.

REFERENCES

1. C. M. ADAMS, Thermal considerations in freezing, *Liquid Metals and Solidification*, pp. 187–217, ASM, Cleveland (1958).
2. F. KREITH, *Principles of Heat Transfer*, p. 597–598, International Textbook Company, Scranton, Pennsylvania (1966).
3. M. JAKOB, *Heat Transfer*, John Wiley, New York (1955).
4. M. EZEKIEL and K. A. FOX, *Methods of Correlation and Regression Analysis*, John Wiley, New York (1959).

Résumé—On présente une méthode pour la mesure de la conductivité thermique d'un liquide à basse température et du coefficient de transport de chaleur entre une sphère de glace en train de se congeler et le liquide à basse température. Les calculs sont basés sur l'égalisation de la quantité de chaleur libérée par une sphère de glace en train de se congeler à la quantité de chaleur qui traverse l'interface glace "moule". En manipulant convenablement les variables directement mesurables, c'est-à-dire le diamètre de la sphère et les temps de congélation, les valeurs de la conductivité thermique et du coefficient de transport de chaleur peuvent être obtenues à partir des pentes et des points d'intersection de lignes droites.

Zusammenfassung—Es wird eine Methode angegeben zur Messung der Wärmeleitfähigkeit einer Flüssigkeit tiever Temperatur und des Wärmeübergangskoeffizienten zwischen einer gefrierenden Eiskugel und der Flüssigkeit tiefer Temperatur. Die Berechnungen beruhen auf einer Bilanz aus freiwerdender Schmelzwärme und der durch die Gefrierzone tretenden Wärmemenge. Durch passende Handhabung der direkt messbaren Veränderlichen wie Kugeldurchmesser und Gefrierzeit können die Werte für Wärmeleitfähigkeit und Wärmeübergangskoeffizient aus Neigungen und Schnittpunkten von geraden Kurven erhalten werden.

Аннотация—Представлен метод измерения теплопроводности низкотемпературной жидкости и коэффициента переноса тепла между шариком льда и низкотемпературной жидкостью. Расчеты основываются на приравнивании количества тепла, освобожденного шариком льда, к количеству тепла, пересекающего границу раздела фаз лед-жидкость. Умело оперируя непосредственно замеряемыми переменными: диаметр шара и время замерзания, можно получить значения теплопроводности и коэффициента переноса тепла по наклонам и пересечениям прямых.